8/11/2019

Estuarine Variability

How much, and how quickly, does an estuary adjust to changes in river flow and tidal mixing?

Review of $\langle$tidally averaged$\rangle$ dynamics:

Decompose into depth-averaged + depth-varying parts:

$$\langle u \rangle = \bar{u} + u' : \quad \langle u \rangle_t \cong -g\langle \eta_x \rangle - g\beta \bar{s}_x z + K u'_{zz}$$

$$\langle s \rangle = \bar{s} + s' : \quad s'_t = -u'\bar{s}_x + K s'_{zz}$$

$$\bar{u} = \frac{Q_R}{A} \qquad \frac{d}{dt}\int s \, dV = \Big(\underbrace{-\bar{u}\bar{s}}_{\text{loss of salt due to } Q_R \cdot \bar{s}} - \underbrace{\overline{u's'}}_{\text{gain of salt due to exchange flow}} \Big) A$$

steady local solutions

$$[u'] = U_E \cong \frac{g\beta \bar{s}_x H}{48}\frac{H^2}{K} = \frac{g\beta s_{ocn} H}{48}\frac{\bar{s}_x}{s_{ocn}}\frac{H^2}{K}$$

so $$[u'] = U_E = \frac{c^2}{48}\underbrace{\left\{\frac{\bar{s}_x}{s_{ocn}}\frac{H^2}{K}\right\}}_{\{\,\}\text{ has units: } \frac{1}{\text{velocity}}} \quad \text{where } c = \sqrt{g\beta s_{ocn} H}$$

and $\frac{[s']}{s_{ocn}} = u_E \frac{\bar{s}_x}{s_{ocn}} \frac{H^2}{K} \sim c^2 \left\{ \frac{\bar{s}_x}{s_{ocn}} \frac{H^2}{K} \right\}^2$

so $\frac{[\overline{u's'}]}{s_{ocn}} \sim c^4 \left\{ \frac{\bar{s}_x}{s_{ocn}} \frac{H^2}{K} \right\}^3$ $\qquad \frac{L^4}{T^4} \frac{1}{L^3} \frac{\cancel{L^6}}{\cancel{L^6}} \cancel{T^3} \sim \frac{L}{t}$ ✓

Note that each of $[u']$, $\frac{[s']}{s_{ocn}}$ and $\frac{[\overline{u's'}]}{s_{ocn}}$ (*)

vary as $\left\{ \frac{\bar{s}_x}{s_{ocn}} \frac{H^2}{K} \right\}$ to some power.

We also showed that the "Chatwin solution"

for $\bar{s}(x)$ to satisfy $-\overline{u's'} - \bar{u}\bar{s} = 0$

gave $\frac{\bar{s}}{s_{ocn}} \sim \left(\frac{x}{L_E} \right)^{3/2}$ where $L_E = 0.024\, c \left(\frac{\bar{u}}{c} \right)^{-1/3} \left(\frac{H^2}{K} \right)$

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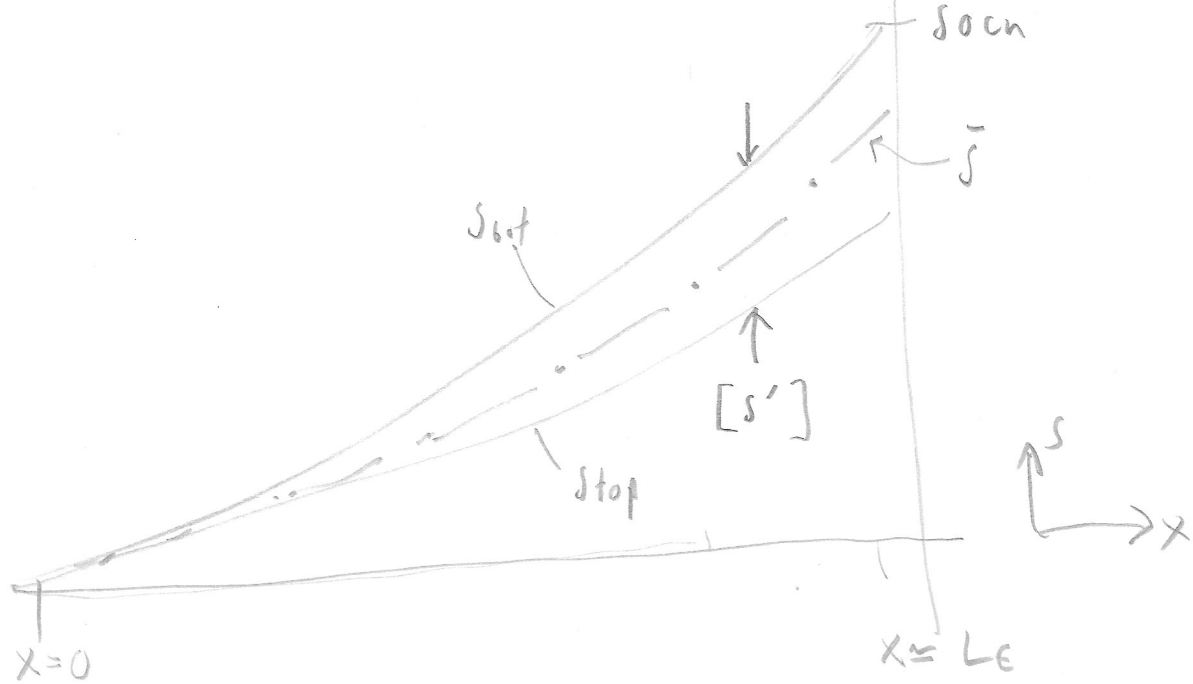
so $\frac{\bar{s}_x}{s_{ocn}} \sim \frac{1}{L_E} \sim \frac{1}{c} \left(\frac{\bar{u}}{c} \right)^{\frac{1}{3}} \frac{K}{H^2} \Rightarrow \frac{\bar{s}_x}{s_{ocn}} \frac{H^2}{K} \sim \frac{1}{c} \left(\frac{\bar{u}}{c} \right)^{\frac{1}{3}}$

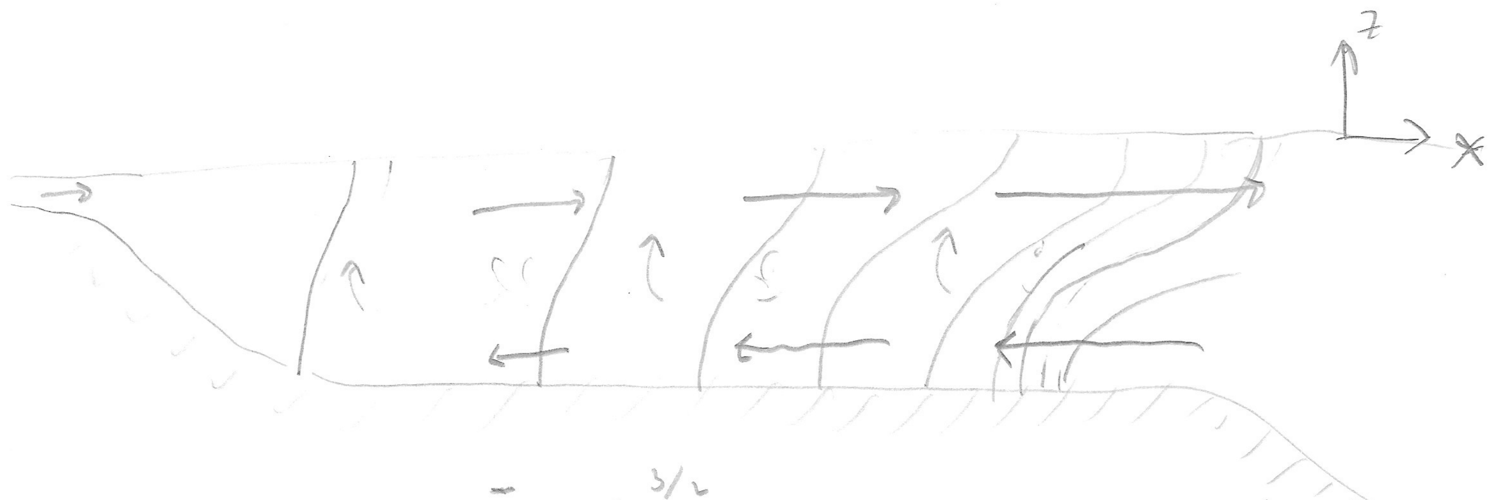
and as a result all the solution fields (*)

are independent of K, and vary weakly with

Q_R $(\sim \bar{u}^{1/3})$

Two representations of the Chatwin solution:





$\bar{s} \sim x^{3/2}$

$s' \sim x$

$u' \sim x^{1/2}$

$$u_E' \sim c^2 \left(\frac{\bar{S}_x}{S_{ocn}} \frac{H^2}{K} \right) \qquad c^2 = g \beta S_{ocn} H$$

$$[S'] \sim c^2 \left(\frac{\bar{S}_x}{S_{ocn}} \frac{H^2}{K} \right)^2$$

$$L_E \sim c \left(\frac{\bar{u}}{c} \right)^{-\frac{1}{3}} \frac{H^2}{K}$$

$\boxed{\text{How do these scale with } H?}$

Variation with H: $c \sim H^{\frac{1}{2}}$, $\bar{u} = \frac{Q_R}{HB} \sim H^{-1}$, $\frac{\bar{u}}{c} \sim H^{-3/2}$

$$\left(\frac{\bar{S}_x}{S_{ocn}} \frac{H^2}{K} \right) \sim \frac{1}{c} \left(\frac{\bar{u}}{c} \right)^{\frac{1}{3}} \sim H^{-\frac{1}{2}} \left(H^{-3/2} \right)^{\frac{1}{3}} \sim H^{-1}$$

so $u_E \sim H H^{-1} \sim$ independent of H

$$[S'] \sim H \left(H^{-2} \right) \sim \frac{1}{H}$$

$$L_E \sim H^{\frac{1}{2}} \left(H^{-3/2} \right)^{-\frac{1}{3}} H^2 = H^{\frac{1}{2}} H^{\frac{1}{2}} H^2 \sim H^3$$

And only L_E varies with K $(\sim \frac{1}{K})$

These results <u>only</u> apply when $\bar{S}_x$ is in equilibrium with the forcing $(\bar{u}, K)$.

But • K varies over the spring Neap cycle (~factor of 2, 2 week period)

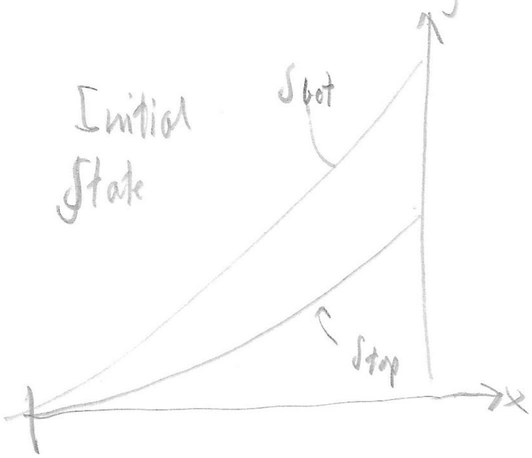
• $\bar{u}$ varies seasonally (and with storms) (~factor 10, 1 year period).

Result: $\bar{S}_x$ often equilibrated to $\bar{u}$, but not to K.

Consider a step change of K

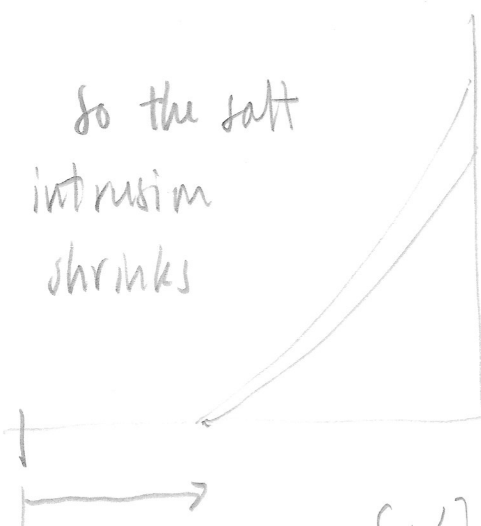
Double K $\Rightarrow$ $[u'] \to \left[\frac{u'}{2}\right]$, $\underbrace{s' \to \left[\frac{s'}{4}\right]}$, $[\overline{u's'}] \to \frac{[\overline{u's'}]}{8}$

(holding $\bar{s}_x$ constant)

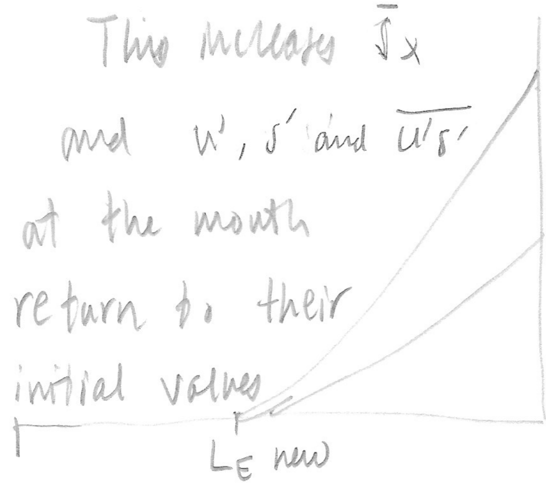


$$\frac{1}{A}\frac{d}{dt}\int_V s\,dV = -\underset{①}{\bar{u}\bar{s}} - \underset{②}{\overline{u's'}}$$ is out of balance

and the system <u>loses salt</u> ([②] is 8x too small)



So the salt intrusion shrinks



This increases $\bar{s}_x$ and u', s' and $\overline{u's'}$ at the mouth return to their initial values

$[u']$: Exchange flow is the same but the residence time is shorter because L_E is shorter.

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How rapidly does the system adjust?

$$\boxed{S} \quad \frac{1}{A}\frac{d}{dt}\underbrace{\int_V S\,dV}_{} = (-\bar{u}\bar{s} - \overline{u's'})A$$

when $K \to K + \Delta K$:

$$\frac{1}{T_{adj}} \frac{\partial}{\partial K}\left(\frac{1}{2} S_{ocn} L_E \cancel{A}\right)\Delta K = (\underbrace{-\bar{u}\bar{s} - \overline{u's'}}_{=0 \text{ initially}})\cancel{A} - \frac{\partial \overline{u's'}}{\partial K}\Delta K \cancel{A}$$

$$\Rightarrow \quad T_{adj} = \frac{\frac{1}{2} S_{ocn}\, \partial L_E/\partial K}{\partial(-\overline{u's'})/\partial K}$$

$$L_E \sim \frac{1}{K} \rightarrow \frac{\partial L_E}{\partial K} \sim -\frac{1}{K^2}$$

$$-\overline{u's'} \sim \frac{1}{K^3} \rightarrow \frac{\partial}{\partial K} -\overline{u's'} \sim -\frac{1}{3}\frac{1}{K^4}$$

$$\Rightarrow \quad T_{adj} \simeq \frac{1}{6}\frac{L_E}{-\overline{u's'}/S_{ocn}} = \frac{1}{6}\frac{L_E}{\bar{u}\bar{s}/S_{ocn}} \sim \boxed{\frac{1}{6}\frac{L_E}{\bar{u}} = T_{adj}}$$

So adjustment time is some fraction of the "freshwater filling time": $L_E/\bar{u}$

Observations suggest $T_{adj} \sim \frac{1}{2}\frac{L_E}{\bar{u}}$ (Lerczak ...